1. Jawaban : A
    Misalkan : p : Masyarakat membuang sampah pada tempatnya.
    q: Kesehatan masyarakat terjaga.
    Diperoleh:
    Premis 1 : $\sim q \Rightarrow \sim p \equiv p \Rightarrow q$
    Premis 2 :   p
    _____

    Kesimpulan :   q

    Jadi, kesimpulan dari premis-premis tersebut adalah "Kesehatan masyarakat terjaga"

2. Jawaban : B
    Misalkan : p: Semua selokan bersih.
    q: Beberapa wilayah bebas nyamuk.
    Pernyataan tersebut dapat ditulis "p ∧ q". $\sim(p \wedge q) \equiv \sim p \vee \sim q$
    Jadi, negasi dari pernyataan tersebut adalah "Beberapa selokan tidak bersih atau semua wilyah tidak bebas nyamuk".

3. Jawaban : B

    $$b^5 \text{ x } \left(\frac{a^{-2}}{c}\right)^4 = b^5 \times \frac{a^{-8}}{c^4}$$

    $$= \frac{b^5}{a^8 c^4}$$

    $$= \frac{3^5}{6^8 \cdot \left(\frac{1}{12}\right)^4}$$

    $$= \frac{3^5}{2^8 \cdot 3^8 \cdot \left(\frac{1}{2^2 \cdot 3}\right)^4}$$

    $$= \frac{3^5}{2^8 \cdot 3^8 \cdot \frac{1}{2^8 \cdot 3^4}} = \frac{3^5}{3^4} = 3$$

4. Jawaban : A

    $$\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+2\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+2\sqrt{2}} \times \frac{\sqrt{5}-2\sqrt{2}}{\sqrt{5}-2\sqrt{2}}$$

    $$= \frac{5-2\sqrt{10}-\sqrt{10}+2\times 2}{5-4\times 2}$$

    $$= \frac{9-3\sqrt{10}}{-3}$$

    $$= \sqrt{10}-3$$

$$= (2 + \sqrt{10}\,) + \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2}} = 2 + \sqrt{10} + \sqrt{10} - 3$$

$$= 2\sqrt{10} - 1$$

5. Jawaban : B

$^3\log 4 = m$

$^5\log 3 = n \Leftrightarrow {}^3\log 5 = \dfrac{1}{n}$

$^{12}\log 45 = \dfrac{^3\log 45}{^3\log 12}$

$$= \dfrac{^3\log(9 \times 5)}{^3\log(4 \times 3)}$$

$$= \dfrac{^3\log 9 + {}^3\log 5}{^3\log 4 + {}^3\log 3}$$

$$= \dfrac{2 + \dfrac{1}{n}}{m + 1} = \dfrac{\dfrac{2n + 1}{n}}{m + 1} = \dfrac{2n + 1}{mn + n}$$

6. **Jawaban : E**

Dari persamaan $x^2 - (m + 1)x + (2m - 2) = 0$ diperoleh :

$$x_1 + x_2 = -\frac{b}{a} = m + 1$$

$$x_1 \cdot x_2 = \frac{c}{a} = 2m - 2$$

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2$$

$\Leftrightarrow \qquad\qquad 20 = (m + 1)^2 - 2(2m - 2)$

$\Leftrightarrow \qquad\qquad 20 = m^2 + 2m + 1 - 4m + 4$

$\Leftrightarrow \qquad\qquad 20 = m^2 - 2m + 5$

$\Leftrightarrow m^2 - 2m - 15 = 0$

$\Leftrightarrow (m - 5)(m + 3) = 0$

$\Leftrightarrow m = 5$ atau $m = -3$

Jadi, nilai $m = -3$ atau $m = 5$

7. **Jawaban : B**

Dari persamaan

$10x^2 - (4m + 4)x + (2m + 2) = 0$ diperoleh:

$a = 10, b = -4m - 4, c = 2m + 2$

$D = b^2 - 4ac$

$\quad = (-4m - 4)^2 - 4 \cdot 10 \cdot (2m + 2)$

$\quad = (16m^2 + 32m + 16) - 80m - 80$

$\quad = 16m^2 - 48m - 64$

$\quad = 16(m^2 - 3m - 4)$

$\quad = 16(m - 4)(m + 1)$

Persamaan kuadrat mempunyai 2 akar real jika D > 0 sehingga:
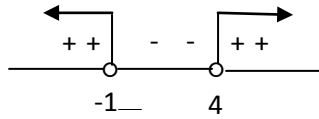
D > 0 $\Leftrightarrow$ 16(m – 4)(m + 1) > 0

Pembuat nol:

16(m – 4)(m + 1) = 0

$\Leftrightarrow$ m – 4 = 0 atau m + 1 = 0

$\Leftrightarrow$ m = 4 atau m = -1

Jadi, nilai m yang memenuhi adalah m < -1 atau m > 4

8. **Jawaban : B**

Misalkan: x = banyak uang Kikan

           y = banyak uang Lusi

           z = banyak uang Maman

Diperoleh system persamaan linear sebagai berikut:

x + y = 32.000   .... (1)

y + z = 38.000   .... (2)

x + y + z = 52.000   .... (3)

Jumlahkan (1) dan (2) :

$$
\begin{array}{r}
x + y = 32.000 \\
\underline{y + z = 38.000} \\
x + 2y + = 70.000
\end{array} + \quad ... (4)
$$

Kurangkan (3) dari (4):

$$
\begin{array}{r}
x + 2y + z = 70.000 \\
\underline{x + y + z = 52.000} \\
y = 18.\,000
\end{array} -
$$

y = 18.000 $\Rightarrow$ x + y + z = 52.000

               $\Leftrightarrow$ x +18.000 + z = 52.000

               $\Leftrightarrow$ x + z = 34.000

Jadi, jumlah uang Kikan dan Maman Rp. 34.000,00.

9. **Jawaban : D**

$L_1 \equiv x^2 + y^2 - px - 8y - 39 = 0$ melalui (11,4)

$\Leftrightarrow 11^2 + 4^2 - p(11) - 8(4) - 39 = 0$

$\Leftrightarrow$ $121 + 16 - 11p - 32 - 39 = 0$

$\Leftrightarrow$ $-11p + 66 = 0$

$\Leftrightarrow$ $-11p = -66$

$\Leftrightarrow$ $p = 6$

$L_1 \equiv x^2 + y^2 - 6x - 8y - 39 = 0$

$\Leftrightarrow (x - 3)^2 - 9 + (y - 4)^2 - 16 - 39 = 0$

$\Leftrightarrow$ $(x - 3)^2 + (y - 4)^2 = 64$

$\Leftrightarrow$ $(x - 3)^2 + (y - 4)^2 = 8^2$

Lingkaran L_1 berpusat di (3,4) dan berjari-jari 8. Lingkaran L_2 berpusat di (3, 4) dan berjari-jari 16. Persamaan lingkaran L_2:

$(x - 3)^2 + (y - 4)^2 = 16^2$

$\Leftrightarrow x^2 - 6x + 9 + y^2 - 8y + 16 - 256 = 0$

$\Leftrightarrow \quad x^2 + y^2 - 6x - 8y - 231 = 0$

Jadi, persamaan lingkaran L_2 adalah $x^2 + y^2 - 6x - 8y - 231 = 0$.

10. **Jawaban: C**

$f(x)$ dibagi $(x^3 - 5x^2 + 4x)$ bersisa $2x^2 + 6x \Leftrightarrow f(x)$ dibagi $x(x - 1)(x - 4)$ bersisa $2x^2 + 6x$

Berdasarkan teorema sisa, diperoleh:

$f(0) = 2(0)^2 + 6(0) = 0$

$f(1) = 2(1)^2 + 6(1) = 8$

$f(4) = 2(4)^2 + 6(4) = 56$

$f(x)$ dibagi $x^2 - 5x - 6$ bersisa $56x + 72$.

Misal: hasil baginya $H(x) = ax^2 + bx + c$

$f(x) = (x^2 - 5x - 6)(ax^2 + bx + c) + 56x + 72$

$f(0) = (0^2 - 5(0) - 6)(a(0)^2 + b(0) + c) + 56(0) + 72$

$\Leftrightarrow 0 = (-6)(c) + 72$

$\Leftrightarrow c = \dfrac{-72}{-6} = 12 \quad \ldots (1)$

$f(1) = (1^2 - 5(1) - 6)(a + b + c) + 56(1) + 72$

$\Leftrightarrow \quad 8 = (-10)(a + b + c) + 128$

$\Leftrightarrow -120 = (-10)(a + b + c)$

$\Leftrightarrow \quad 12 = a + b + c \quad \ldots (2)$

$f(4) = (4^2 - 5(4) - 6)(a + b + c) + 56(4) + 72$

$\Leftrightarrow \quad 56 = (-10)(16a + 4b + c) + 296$

$\Leftrightarrow -240 = (-10)(16a + 4b + c)$

$\Leftrightarrow \quad 24 = 16a + 4b + c \quad \ldots (3)$

Substitusi $c = 12$ ke (2) dan (3):

$c = 12 \quad \Leftrightarrow \quad 12 = a + b + 12$

$\qquad\qquad \Leftrightarrow a + b = 0 \quad \ldots (4)$

$c = 12 \quad \Leftrightarrow \qquad 24 = 16a + 4b + 12$

$\qquad\qquad \Leftrightarrow 16a = 4b = 12 \quad \ldots (5)$

Eliminasi b dari (4) dan (5):

$\begin{array}{r} a + b = 0 \\ 16a + 4b = 12 \end{array} \left| \begin{array}{l} \times 4 \\ \times 1 \end{array} \right| \begin{array}{r} 4a + 4b = 0 \\ 16a + 4b = 12 \end{array}$

$\qquad\qquad\qquad\qquad\qquad \overline{\qquad\qquad\qquad} \, -$

$\qquad\qquad\qquad\qquad\qquad -12a = -12$

$\qquad\qquad\qquad\qquad\qquad\quad a = 1$

Substitusi $a = 1$ ke $a + b = 0$

$\Leftrightarrow 1 + b = 0 \Leftrightarrow b = -1$

Diperoleh $a = 1$, $b = -1$, dan $c = 12$.

Jadi, hasil baginya $H(x) = ax^2 + bx + c$

$\qquad\qquad\qquad\qquad\quad = x^2 - x + 12$

11. **Jawaban : D**

$(f \circ g)(x) = f(g(x))$

$$= f\left(\frac{x-1}{x+1}\right)$$

$$= \frac{\dfrac{x-1}{x+1} + 2}{\dfrac{x-1}{x+1} - 2}$$

$$= \frac{\dfrac{x-1+2(x+1)}{x+1}}{\dfrac{x-1-2(x+1)}{x+1}}$$

$$= \frac{x-1+2x+2}{x-1-2x-2}$$

$$= \frac{3x+1}{-x-3}$$

$(f \circ g)(-2) = \dfrac{3(-2)+1}{-(-2)-3} = \dfrac{-5}{-1} = 5$

12. **Jawaban : E**

Misalkan

x = banyak makanan jenis A

b = banyak makanan jenis B

Makanan	Protein	Karbohidrat	Lemak	Harga
Jenis A (x)	2	6	1	10.000
Jenis B (y)	1	1	3	8.000
Kendala	8	12	9	

Diperoleh SPtLDV :

$2x + y \geq 8$

$6x + y \geq 12$

$x + 3y \geq 9$

$x \geq 0$

$y \geq 0$

Fungsi objektif: $f(x, y) = 10.000x + 8.000y$

Daerah penyelesaian SPtLDV:
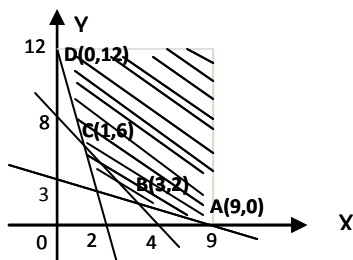
Uji titik pojok ke fungsi objektif:

Titik Pojok	$f(x,y) = 10.000x + 8.000y$
A(9, 0)	10.000 x 9 + 8.000 x 0 = 90.000
B(3, 2)	10.000 x 3 + 8.000 x 2 = 46.000
C(1, 6)	10.000 x 1 + 8.000 x 6 = 58.000
D(0, 12)	10.000 x 0 + 8.000 x 12 = 96.000

Nilai minimum f(x, y) adalah 46.000.
Jadi, uang yang harus di keluarkan minimum Rp. 46.000,00

13. **Jawaban : C**

$3A - 2B = C$

$$\Leftrightarrow \quad 3\begin{pmatrix} a & 3 \\ 4 & b \end{pmatrix} - 2\begin{pmatrix} b & 5 \\ 4 & a \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 4 & -4 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 3a & 9 \\ 12 & 3b \end{pmatrix} + \begin{pmatrix} -2b & -10 \\ -8 & -2a \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 4 & -4 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 3a - 2b & -1 \\ 4 & 3b - 2a \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 4 & -4 \end{pmatrix}$$

Dari kesamaan matriks, diperoleh:

$$
\begin{array}{l}
3a - 2b = 1 \quad \left| \begin{array}{l} \times\,2 \\ \times\,3 \end{array} \right| \quad \begin{array}{r} 6a - 4b = 2 \\ -6a + 9b = -12 \, + \\ \hline 5b = -10 \end{array} \\
-2a + 3b = -4
\end{array}
$$

$$\Leftrightarrow \quad b = -2$$

Substitusikan b = -2 ke 3a − 2b = 1.
3a − 2(-2) = 1
$\Leftrightarrow \quad 3a = 1 - 4$
$\Leftrightarrow \quad 3a = -3$
$\Leftrightarrow \quad a = -1$
Diperoleh a = -1 dan b = -2
Nilai a + b = -1 + (-2) = -3.

14. **Jawaban : C**

Panjang vektor $\vec{a} = 2\vec{i} + x\vec{j} - 2x\vec{k}$ adalah 3 satuan.

$|\vec{a}| = 3$

$\Leftrightarrow \sqrt{2^2 + x^2 + (-2x)^2} = 3$

$\Leftrightarrow \sqrt{4 + 5x^2} = 3$

$\Leftrightarrow 4 + 5x^2 = 9$

$\Leftrightarrow 5x^2 = 5$

$\Leftrightarrow x^2 = 1$

$\Leftrightarrow x = \pm 1$

Oleh karena x < 0 maka x = -1

$\vec{a} = 2\vec{i} + x\vec{j} - 2x\vec{k} = 2\vec{i} - \vec{j} + 2\vec{k}$

Vektor $\vec{b}$ tegak lurus vektor $\vec{c}$, maka berlaku : $\vec{b} \cdot \vec{c} = 0$

$\Leftrightarrow (-6) \times 2 + 1 \times y + (-2) \times (-3) = 0$

$\Leftrightarrow -12 + y + 6 = 0$

$\Leftrightarrow y = 6$

$\vec{c} = 2\vec{i} + y\vec{j} - 3\vec{k} = 2\vec{i} + 6\vec{j} - 3\vec{k}$

$\vec{a} \cdot \vec{c} = 2 \times 2 + (-1) \times 6 + 2 \times (-3)$

$\qquad = 4 - 6 - 6$

$\qquad = \text{-8}$

Jadi, $\vec{a} \cdot \vec{c} = -8$

15. **Jawaban : D**

$$\vec{BA} = \vec{a} - \vec{b} = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{AC} = \vec{c} - \vec{a} = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$$

Misal $^\theta$ = sudut antara vektor $\vec{BA}$ dan $\vec{AC}$.

$$\cos{^\theta} = \frac{\vec{BA} \cdot \vec{AC}}{\left| \vec{BA} \right| \cdot \left| \vec{AC} \right|}$$

$$= \frac{0 \times (-2) + (-1) \times 2 + 1 \times (-1)}{\sqrt{0^2 + (-1)^2 + 1^2} \cdot \sqrt{(-2)^2 + 2^2 + (-1)^2}}$$

$$= \frac{0 - 2 - 1}{\sqrt{2} \cdot \sqrt{9}}$$

$$= \frac{-3}{3\sqrt{2}} = -\frac{1}{2}\sqrt{2}$$

Oleh karena $\cos{^\theta} = -\dfrac{1}{2}\sqrt{2}$ maka $^\theta = 135^0$.

Jadi, besar sudut antara vektor $\vec{BA}$ dan $\vec{AC}$ adalah 135^0.

16. **Jawaban : E**

$\vec{W}$ = proyeksi orthogonal vektor $\vec{v}$ pada vektor $\vec{u}$

$$\vec{W} = \frac{\vec{v} \cdot \vec{u}}{\left| \vec{u} \right|^2} \cdot \vec{u}$$

$$= \frac{2 \times (-1) + (-3) \times 2 + 4 \times (-1)}{(\sqrt{(-1)^2 + 2^2 + (-1)^2})^2} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

$$= \frac{-2-6-4}{(\sqrt{6})^2}\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

$$= \frac{-12}{6}\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = -2\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}$$

Jadi, $\vec{w} = \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}$

17. **Jawaban : B**

$$(x,y) \xrightarrow{M_{y=x}} (y,x) \xrightarrow[R(O,\,-90^0)]{R(O,\,270^{0+})} (x,y) \xrightarrow{M_y} (-x,\,-y)$$

Diperoleh :
$x' = -x \Leftrightarrow x = -x'$
$y' = -y \Leftrightarrow y = -y'$
substitusikan x dan y ke persamaan garis :
$3x + 2y = 15$

$\Leftrightarrow 3(-x') + 2(-y') = 15$

$\Leftrightarrow -3x' - 2y' = 15$

$\Leftrightarrow 3x' + 2y' = -15$

Jadi, persamaan bayangannya 3x + 2y = -15.

18. **Jawaban : D**

$$\frac{27^{4-2x}}{3\sqrt{3}} \leq (9\sqrt{3})^{x+2}$$

$$\Leftrightarrow \frac{3^{3(4-2x)}}{3^{1\frac{1}{2}}} \leq (3^{2\frac{1}{2}})^{x+2}$$

$$\Leftrightarrow 3^{3(4-2x)-1\frac{1}{2}} \leq 3^{2\frac{1}{2}(x+2)}$$

$$\Leftrightarrow 3(4-2x) - 1\frac{1}{2} \leq 2\frac{1}{2}(x+2)$$

$$\Leftrightarrow 12 - 6x - \frac{3}{2} \leq \frac{5}{2}x + 5$$

$$\Leftrightarrow -6x - \frac{5}{2}x \leq 5 - 12 + \frac{3}{2}$$

$$\Leftrightarrow -\frac{17}{2}x \leq -\frac{11}{2}$$

$$\Leftrightarrow x \geq \frac{11}{17}$$

Jadi, himpunan penyelesaiannya $\{x \mid x \geq \frac{11}{17}\}$.

19. **Jawaban : A**

Grafik fungsi melalui titik (-2, -1), (0, 0), dan (6, 1). Fungsi yang memenuhi adalah f(x) = $^3\log(x+3) - 1$ karena :

$f(-2) = {}^3\log(-2+3) - 1 = 0 - 1 = -1$
$f(0) = {}^3\log(0+3) - 1 = 1 - 1 = 0$
$f(6) = {}^3\log(6+3) - 1 = 2 - 1 = 1$

20. **Jawaban : C**

Deret aritmetika : $U_n = a + (n-1)b$

$U_3 + U_5 + U_7 = 12$

$\Leftrightarrow a + 2b + a + 4b + a + 6b = 12$

$\Leftrightarrow 3a + 12b = 12$

$\Leftrightarrow a + 4b = 4$ . . . (1)

$U_4 + U_8 = -U_6$

$\Leftrightarrow U_4 + U_6 + U_8 = 0$

$\Leftrightarrow a + 3b + a + 5b + a + 7b = 0$

$\Leftrightarrow 3a + 15b = 0$

$\Leftrightarrow a + 5b = 0$ . . . (2)

Eliminasi a dari (1) dan (2):

$a + 4b = 4$
$\underline{a + 5b = 0\ -}$
$\quad\ -\ \ b\ = 4$

$\Leftrightarrow b = -4$

Substitusi b = -4 ke persamaan (1):

$a + 4b = 4 \Rightarrow a + 4(-4) = 4$

$\qquad\qquad \Leftrightarrow a = 4 + 16$
$\qquad\qquad \Leftrightarrow a = 20$

Diperoleh a = 20 dan b = -4

Jumlah dua belas suku pertama :

$S_{12} = \dfrac{12}{2}(2a + (12-1)b)$

$\quad = 6\,(2(20) + 11(-4))$
$\quad = 6\,(40 - 44)$
$\quad = 6\,(-4)$
$\quad = -24$

21. **Jawaban : B**

Pengambilan uang mengikuti aturan deret aritmetika.

Bulan I diambil: $U_1 = $ Rp. 800.000,00
Bulan II diambil: $U_2 = $ Rp. 775.000,00
Bulan III diambil: $U_3 = $ Rp. 750.000,00

Diperoleh: a = 800.000
$\qquad\qquad$ b = 775.000 − 800.000 = - 25.000

$S_{12} = \dfrac{12}{2}(2a + (12-1)b)$

$\quad = 6\,(2(800.000) + 11(-25.000))$
$\quad = 6\,(1.600.000 - 275.000)$
$\quad = 6(1.325.000)$

$$= 7.950.000$$

Jadi, jumlah pengambilan uang selama 12 bulan pertama Rp. 7. 950.000,00.

22. **Jawaban : C**

Deret geometri : $U_2 = 54$ dan $U_5 = 16$

$$\frac{U_5}{U_2} = \frac{ar^4}{ar} = \frac{16}{54}$$

$$\Leftrightarrow r^3 = \frac{8}{27}$$

$$\Leftrightarrow r = \frac{2}{3}$$

$$U_2 = 54 \Leftrightarrow a \times \frac{2}{3} = 54$$

$$\Leftrightarrow a = 54 \times \frac{3}{2} = 81$$

Jumlah semua suku :

$$S_\infty = \frac{a}{1-r} = \frac{81}{1-\frac{2}{3}} = \frac{81}{\frac{1}{3}} = 243$$

23. **Jawaban : A**

Panjang potongan kawat membentuk barisan geometri.

$U_1 = 15 \Leftrightarrow a = 15$

$U_6 = 480 \Leftrightarrow 15r^5 = 480$

$$\Leftrightarrow r^5 = 32$$

$$\Leftrightarrow r = 2$$

Jumlah enam suku pertama :

$$S_6 = \frac{a(r^6 - 1)}{r - 1} = \frac{15(2^6 - 1)}{2 - 1} = \frac{15(64 - 1)}{1}$$

$$= 15(63)$$

$$= 945 \text{ cm.}$$

24. **Jawaban : D**
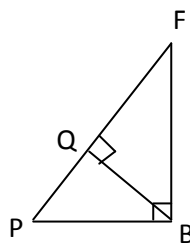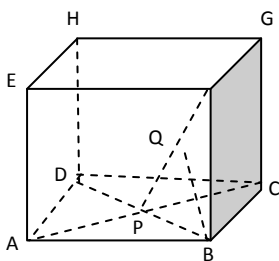
Jarak antara titik B ke Bidang ACF sama dengan jarak antara titik B ke garis PF dengan P titik tengah AC, yaitu sama dengan panjang BQ. BD dan AC merupakan diagonal sisi maka panjang BD = AC = $6\sqrt{2}$ cm.

Segitiga PBF siku-siku di B:

BF = 6 cm

$PB = \dfrac{1}{2}BD = \dfrac{1}{2} \times 6\sqrt{2} = 3\sqrt{2}\,cm$

$PF = \sqrt{PB^2 + BF^2}$

$\quad = \sqrt{18+36}$

$\quad = \sqrt{54}$

$\quad = 3\sqrt{6}$ cm

Perhatikan segitiga PBF.

Misalkan PQ = x cm, maka FQ = $(3\sqrt{6} - x)cm$.

$BQ^2 = BP^2 - PQ^2 = BF^2 - FQ^2$

$\Leftrightarrow (3\sqrt{2})^2 - x^2 = 6^2 - (3\sqrt{6} - x)^2$

$\Leftrightarrow 18 - x^2 = 36 - 54 + 6\sqrt{6}x - x^2$

$\Leftrightarrow 36 = 6\sqrt{6}x$

$\Leftrightarrow x = \dfrac{36}{6\sqrt{6}} = \sqrt{6}$

Diperoleh panjang PQ = $\sqrt{6}$ cm.

$BQ = \sqrt{BP^2 - PQ^2}$

$\quad = \sqrt{18-6} = \sqrt{12} = 2\sqrt{3}\,cm$.

Jadi, jarak titik B ke bidang ACF adalah $2\sqrt{3}\,cm$.

25. **Jawaban : A**

Proyeksi PQ pada ABCD adalah PC, maka sudut antara PQ dan ABCD sama dengan $\angle QPC$. Segitiga PBC siku-siku di B, maka :
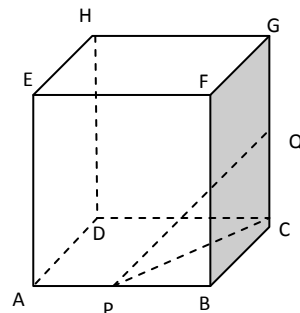
$PC = \sqrt{PB^2 + BC^2}$

$\quad = \sqrt{\left(\dfrac{3}{2}\right)^2 + 3^2} = \sqrt{\dfrac{9}{4}+9} = \sqrt{\dfrac{45}{4}} = \dfrac{3}{2}\sqrt{5}\,cm$

Segitiga PCQ siku-siku di C, maka :

$PQ = \sqrt{PC^2 + CQ^2}$

$\quad = \sqrt{\left(\tfrac{3}{2}\sqrt{5}\right)^2 + \left(\tfrac{3}{2}\right)^2}$

$\quad = \sqrt{\tfrac{45}{4} + \tfrac{9}{4}} = \sqrt{\tfrac{54}{4}} = \tfrac{3}{2}\sqrt{6}\,cm$
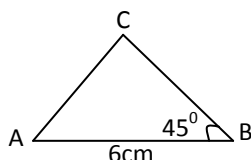
$\sin \angle QPC = \dfrac{CQ}{PQ} = \dfrac{\tfrac{3}{2}}{\tfrac{3}{2}\sqrt{6}} = \tfrac{1}{6}\sqrt{6}$

Jadi, nilai sinus sudut antara garis PQ dan bidang ABCD adalah $\dfrac{1}{6}\sqrt{6}$.

26. **Jawaban : E**

Perhatikan alas limas berikut.

Luas alas = L_{ABC}

$$= \frac{1}{2} \times AB \times BC \times \sin \angle ABC$$

$$= \frac{1}{2} \times 6 \times 6 \times \sin 45^0$$

$$= \frac{1}{2} \times 6 \times 6 \times \frac{1}{2}\sqrt{2}$$

$$= 9\sqrt{2}\,cm^2$$

Volume limas $= \frac{1}{3}$ x luas alas x tinggi

$$= \frac{1}{3} \times 9\sqrt{2} \times 15$$

$$= 45\sqrt{2}\,cm^3$$

Jadi, volume limas tersebut $45\sqrt{2}\,cm^3$.

27. **Jawaban : D**

   $\sin (2x + 60^0) - \cos (x + 30^0) = 0$

   $\Leftrightarrow \sin 2(x + 30^0) + \cos(x + 30^0) = 0$

   $\Leftrightarrow 2\sin(x + 30^0)\cos(x + 30^0) - \cos(x + 30^0) = 0$

   $\Leftrightarrow \cos(x + 30^0)(2\sin(x + 30^0) - 1) = 0$

   $\Leftrightarrow \cos(x + 30^0) = 0$ atau $\sin(x + 30^0) = \frac{1}{2}$

   a. $\cos(x + 30^0) = 0 \cos 90^0$
      Penyelesaiannya :
      $x + 30^0 = 90^0 + k \cdot 360^0$
      $\Leftrightarrow \quad x = 60^0 + k \cdot 360^0$
      Untuk k = 0, maka x = 60^0

   b. $\sin(x + 30^0) = \frac{1}{2} = \sin 30^0$

      penyelesaiannya :
      1) $x + 30^0 = 30^0 + k \cdot 360^0$
         $\Leftrightarrow x = k \cdot 360^0$
         Untuk k = 0, maka x = 0^0
      2) $x + 30^0 = (180^0 - 30^0) + k \cdot 360^0$
         $\Leftrightarrow x = k \cdot 360^0$
         $\Leftrightarrow x = 120^0 + k \cdot 360^0$
         Untuk k = 0, maka x = 120^0
   Jadi, penyelesaiannya $\{0^0, 60^0, 120^0\}$.

28. **Jawaban : D**

$$\frac{\sin(x + 100)^0 - \sin(x + 40)^0}{\cos(x + 100)^0 + \cos(x + 40)^0}$$

$$= \frac{2 \cos \frac{1}{2}(2x + 140)^0 \sin \frac{1}{2}(60)^0}{2 \cos \frac{1}{2}(2x + 140)^0 \cos \frac{1}{2}(60)^0}$$

$$= \frac{2 \cos(x + 70)^0 \sin 30^0}{2 \cos(x + 70)^0 \cos 30^0}$$

$$= \frac{\sin 30^0}{\cos 30^0}$$

$$= \tan 30^0 = \frac{1}{3}\sqrt{3}$$

29. **Jawaban : A**
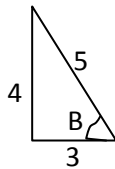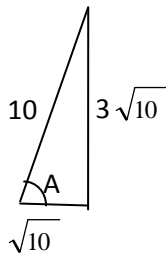
$$\sin A = \frac{3}{10}\sqrt{10} \qquad\qquad \cos B = -\frac{3}{5}$$

$$\tan A = 3 \qquad\qquad\qquad \tan B = -\frac{1}{3}$$

$$\tan(2A + 2B) = \tan 2(A + B)$$

$$= \frac{2 \tan(A + B)}{1 - \tan^2(A + B)}$$

Mencari nilai tan (A + B) terlebih dahulu

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{3 + \left(-\frac{4}{3}\right)}{1 - 3\left(-\frac{4}{3}\right)} = \frac{\frac{5}{3}}{5} = \frac{1}{3}$$

$$\tan(2A + 2B) = \frac{2 \tan(A + B)}{1 - \tan^2(A + B)}$$

$$= \frac{2\left(\frac{1}{3}\right)}{1 - \left(\frac{1}{3}\right)^2}$$

$$= \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{2}{3} \times \frac{9}{8} = \frac{3}{4}$$

Jadi, nilai tan (2A + 2B) = $\frac{3}{4}$

30. **Jawaban : E**

$$\lim_{x \to \infty} x^2 - x\sqrt{x^2 - 5}$$

$$= \lim_{x \to \infty} x^2 - x\sqrt{x^2 - 5} \times \frac{x^2 + x\sqrt{x^2 - 5}}{x^2 + x\sqrt{x^2 - 5}}$$

$$= \lim_{x \to \infty} \frac{x^4 - x^2(x^2 - 5)}{x^2 + x\sqrt{x^2 - 5}}$$

$$= \lim_{x \to \infty} \frac{x^4 - x^4 + 5x^2}{x^2 + x\sqrt{x^2 - 5}}$$

$$= \lim_{x \to \infty} \frac{5x^2}{x^2 + x\sqrt{x^2 - 5}}$$

$$= \lim_{x \to \infty} \frac{5}{x^2 + x\sqrt{1 - \frac{5}{x^2}}}$$

$$= \frac{5}{1 + \sqrt{1 - 0}} = \frac{5}{1 + 1} = \frac{5}{2}$$

31. **Jawaban : A**

$$\lim_{x \to 5} \frac{x^2 - 10x + 25}{1 - \cos(4x - 20)}$$

$$= \lim_{x \to 5} \frac{x^2 - 10x + 25}{1 - \cos 2(2x - 10)}$$

$$= \lim_{x \to 5} \frac{x^2 - 10x + 25}{1 - (1 - 2\sin^2(2x - 10))}$$

$$= \lim_{x \to 5} \frac{x^2 - 10x + 25}{2\sin^2(2x - 10)}$$

$$= \lim_{x \to 5} \frac{(x - 5)(x - 5)}{2\sin^2(2x - 10)}$$

$$= \lim_{x \to 5} \frac{x - 5}{2\sin 2(x - 5)} \cdot \lim_{x \to 5} \frac{x - 5}{\sin 2(x - 5)}$$

$$= \frac{1}{2 \cdot 2} \cdot \frac{1}{2} = \frac{1}{8}$$

32. **Jawaban : B**

f(x) = x^3 - 4x^2 + 6

$\Leftrightarrow$ f(2) = $2^3 - 4(2^2) + 6$

$\Leftrightarrow$ f(2) = $8 - 16 + 6$

$\Leftrightarrow$ f(2) = -2

Titik singgung (2, -2)

Gradient garis singgung: m = f'(x) = $3x^2 - 8x$

Substitusikan x = 2 ke m :

m = $3x^2 - 8x$

$= 3(2^2) - 8(2)$

$= 12 - 16 = -4$

Persamaan garis singgung melalui titik (2, -2) dan bergradien -4 sebagai berikut.

$y - y_1 = m(x - x_1)$

$\Leftrightarrow y + 2 = -4(x - 2)$

$\Leftrightarrow y + 2 = -4x + 8$

$\Leftrightarrow 4x + y = 6$

Jadi, persamaan garis singgungnya $4x + y = 6$.

33. **Jawaban : A**

Integral parsial

Fungsi $3x\sqrt{1-x}$ dapat dipecah menjadi $3x$ dan $\sqrt{1-x} = (1-x)^{\frac{1}{2}}$.

Fungsi $3x$ diturunkan sampai diperoleh nilai nol, sedangkan $(1-x)^{\frac{1}{2}}$ diintegralkan.

Diturunkan   **Diintegralkan**

$3x$ $(1-x)^{\frac{1}{2}}$

        $+$

3 $-\dfrac{2}{3}(1-x)^{\frac{3}{2}}$

        $-$

0 $\dfrac{4}{15}(1-x)^{\frac{5}{2}}$

$\int 3x\sqrt{1-x}\, dx$

$= 3x \cdot (-\dfrac{2}{3}(1-x)^{\frac{3}{2}}) - 3 \cdot \dfrac{4}{15}(1-x)^{\frac{5}{2}} + C$

$= -2x(1-x)^{\frac{3}{2}} - \dfrac{4}{5}(1-x)^{\frac{5}{2}} + C$

$= -2(1-x)^{\frac{3}{2}}(x + \dfrac{2}{5}(1-x)) + C$

$= -2(1-x)^{\frac{3}{2}}(x + \dfrac{2}{5} - \dfrac{2}{5}x) + C$

$= -\dfrac{2}{5}(1-x)^{\frac{3}{2}}(3x + 2) + C$

$= -\dfrac{2}{5}\sqrt{(1-x)^3}(3x + 2) + C$

$= -\dfrac{2}{5}(3x + 2)\sqrt{(1-x)^3} + C$

34. **Jawaban : C**
    Integral parsial
    Fungsi $(x + 2)(2x - 1)^4$ dapat dipecah menjadi $(x + 2)$ dan $(2x - 1)^4$.
    fungsi $(x + 2)$ diturunkan sampai diperoleh nilai nol, sedangkan $(2x - 1)^4$ diintegralkan.

Diturunkan	**Diintegralkan**
$x + 2$	$(2x = 1)^4$
1	$\dfrac{1}{10}(2x - 1)^5$
0	$\dfrac{1}{120}(2x - 1)^6$

$$\int_0^1 (x + 2)(2x - 1)^4 \, dx$$

$$= \left[\frac{1}{10}(x + 2)(2x - 1)^5 - \frac{1}{120}(2x - 1)^6\right]_0^1$$

$$= \left(\frac{1}{10}(1 + 2)(2 - 1)^5 - \frac{1}{120}(2 - 1)^6\right) - \left(\frac{1}{10}(0 + 2)(0 - 1)^5 - \frac{1}{120}(0 - 1)^6\right)$$

$$= \frac{3}{10} - \frac{1}{120} + \frac{2}{10} + \frac{1}{120}$$

$$= \frac{5}{10} = \frac{1}{2}$$

35. Jawaban : D

$$\int_{-\pi}^{0} (\cos x - \sin x)^2 \, dx$$

$$= \int_{-\pi}^{0} (\cos^2 x - 2\sin x \cos x + \sin^2 x) \, dx$$

$$= \int_{-\pi}^{0} (1 - \sin 2x) \, dx$$
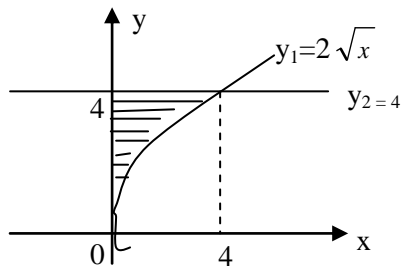
$$= \left[x + \frac{1}{2}\cos 2x\right]_{-\pi}^{0}$$

$$= 0 + \frac{1}{2}\cos 0 - \left(-\pi + \frac{1}{2}\cos(-2\pi)\right)$$

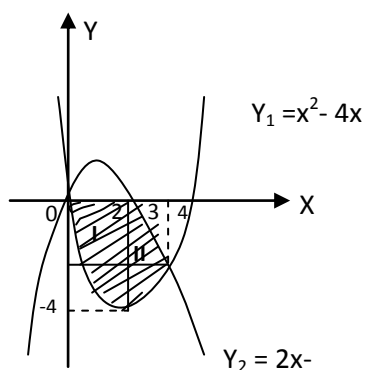$$= \frac{1}{2} \cdot 1 + \pi - \frac{1}{2} \cdot 1 = \pi$$

36. **Jawaban : B**

$y^2 = 4x \Leftrightarrow y = \pm 2\sqrt{x}$

$L = \int\limits_{0}^{4} (y_2 - y_1)\, dx$

$= \int\limits_{0}^{4} (4 - 2\sqrt{x})\, dx$

$= \int\limits_{0}^{4} (4 - 2x^{\frac{1}{2}})\, dx$

$= \left[4x - \frac{2}{\frac{3}{2}} x^{\frac{3}{2}} \right]_{0}^{4}$

$= 4(4 - 0) - \frac{4}{3}(4^{\frac{3}{2}} - 0^{\frac{3}{2}})$

$= 4 \cdot 4 - \frac{4}{3}(2^2)^{\frac{3}{2}}$

$= 16 - \frac{4}{3} \cdot 2^3$

$= 16 - \frac{32}{3}$

$= 5\frac{1}{3}$ satuan luas

37. **Jawaban : E**

Daerah yang diarsir terbagi menjadi daerah I dan daerah II.
Daerah I dibatasi kurva $y = x^2 - 4x$, sumbu X, dan garis x = 2.
Daerah II dibatasi kurva $y = 2x - x^2$, $y = x^2 - 4x$, dan garis x = 2.

$V = V_I + V_{II}$

$$= \pi \int_0^2 y_1{}^2 \, dx + \pi \int_2^3 (y_1{}^2 - y_2{}^2) \, dx$$

$$= \pi \int_0^2 (x^2 - 4x)^2 \, dx + \pi \int_2^3 ((x^2 - 4x)^2 - (2x - x^2)^2) \, dx$$

$$= \pi \int_0^2 (x^4 - 8x^3 + 16x^2) \, dx + \pi \int_2^3 (x^4 - 8x^3 + 16x^2) - (4x^2 - 4x^3 + x^4)) \, dx$$

$$= \pi \int_0^2 (x^4 - 8x^3 + 16x^2) \, dx + \pi \int_2^3 (12x^2 - 4x^3) \, dx$$

$$= \pi \left[\frac{1}{5}x^5 - 2x^4 + \frac{16}{3}x^3 \right]_0^2 + \pi \left[4x^3 - x^4 \right]_2^3$$

$$= \pi \left(\frac{1}{5}(2^5 - 0^5) + 2(2^4 - 0^4) + \frac{16}{3}(2^3 - 0^3) \right) + \pi (4(3^3 - 2^3) - (3^4 - 2^4))$$

$$= \pi \left(\frac{1}{5} \cdot 32 - 2 \cdot 16 + \frac{16}{3} \cdot 8 \right) + \pi (4 \cdot 19 - 65)$$

$$= 32\pi \left(\frac{1}{5} - 1 + \frac{4}{3} \right) + \pi (76 - 65)$$

$$= \frac{226}{15}\pi + 11\pi$$

$$= 17\frac{1}{15}\pi + 11\pi$$

$$= 28\frac{1}{15}\pi \text{ satuan volume}$$

38. **Jawaban : B**

Banyak cara memilih siswa putra yang duduk di kursi pinggir = $_5C_2$

Banyak cara duduk dua siswa putra di pinggir = 2!

Tiga siswa putri selalu duduk berdampingan maka dianggap satu unsur sehingga banyak siswa yang duduk di tengah tinggal 4 orang.

Banyak cara duduk 4 orang siswa di tengah = 4!

Banyak cara duduk 3 siswa putri yang selalu berdampingan = 3!

Banyak posisi duduk = $_5C_2 \cdot 2!3!4! = 10 \cdot 2 \cdot 6 \cdot 24 = 2.880$

39. **Jawaban : C**

Titik Tengah Data (x_i)	f_i	$f_i x_i$
$\frac{1}{2}(0,5 + 5,5) = 3$	$30 - 25 = 5$	15
$\frac{1}{2}(5,5 + 10,5) = 8$	$25 - 22 = 3$	24
$\frac{1}{2}(10,5 + 15,5) = 13$	$22 - 14 = 8$	104
$\frac{1}{2}(15,5 + 20,5) = 18$	$14 - 8 = 6$	108
$\frac{1}{2}(20,5 + 25,5) = 23$	8	184
	$\sum f_i$	$\sum f_i x_i = 435$

Rata-rata panjang potongan pipa :

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{435}{30} = 14,5\,cm$$

40. **Jawaban : B**

S = {1, 2, 3, 4, 5,6}

n(S) = 6

A = kejadian muncul mata dadu genap

= {2, 4, 6}

n(A) = 3

Peluang muncul mata dadu genap:

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

B = kejadian muncul mata dadu prima

= {2, 3, 5}

n(B) = 3

Peluang muncul mata dadu prima:

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Peluang muncul mata dadu genap pada pelemparan pertama dan mata dadu prima pada pelambungan kedua = P(A) x P(B)

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$